REPRESENTATION OF FOURIER SERIES

Fourier series - The representation of signal over a certain interval of time in terms of the linear combination of orthogonal function is called Fourier series. Fourier ~~series~~ series is applicable only for periodic signals.

There are three types of Fourier series (form).

i) Trigonometric form.
ii) Cosine form.
iii) Exponential form.

If the orthogonal functions are trigonometric functions then it is called trigonometric Fourier series.
If the orthogonal functions are exponential function then it is called exponential Fourier series.

Existence of Fourier series:
The condition under which a periodic signal can be represented by a Fourier series are known as Dirichlet's condition.

In each period
1) The function x(t) must be a single valued function
2) The function x(t) has only a finite number of maxima and ~~minima~~ minima.
3) The function x(t) has a finite number of discontinuities.
4) The function x(t) is absolutely integrable over one period i.e

$$\int_0^T |x(t)|\, dt < \infty$$

The condition 4 is also known as the weak of Dirichlet condition.
The conditions 2 and 3 are known as strong Dirichlet conditions.

* If the function (periodic signal) satisfies the weak Dirichlet condition, the existence of Fourier series is guaranteed but the series may not converge at every point. For strong condition satisfied, the convergence is also guaranteed.

TRIGONOMETRIC FORM OF FOURIER SERIES

Let us consider a periodic signal,

$$x(t) = A \sin\omega_0 t \quad \text{with period } T = \frac{2\pi}{\omega_0}$$

Here we can show that a signal $x(t)$, a sum of sine and cosine functions whose frequencies are integral multiples of ω_0 is a periodic signal.
Let the signal $x(t)$ is expressed as,

$$x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots a_k \cos k\omega_0 t + b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \dots + b_k \sin k\omega_0 t.$$

i.e. $$x(t) = a_0 + \sum_{n=1}^{k} a_n \cos\omega_0 n t + b_n \sin n\omega_0 t \quad \text{--- (1)}$$

where $a_0, a_1, a_2 \dots a_k$ and $b_0, b_1, b_2 \dots b_k$ are constants

ω_0 = fundamental frequency

As we know that if a signal is periodic, then it must satisfy the following condition

$$x(t) = x(t+T) \quad \text{for all } T$$

$$\therefore x(t) = x(t+T) = a_0 + \sum_{n=1}^{k} a_n \cos\omega_0 n(t+T) + b_n \sin\omega_0 n(t+T)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos\omega_0 n\left(t + \frac{2\pi}{\omega_0}\right) + b_n \sin\omega_0 n\left(t + \frac{2\pi}{\omega_0}\right)$$

$$\left[\because T = \frac{2\pi}{\omega_0}\right]$$

$$= a_0 + \left[\sum_{n=1}^{k} a_n \cos\left(\omega_0 n t + \frac{2\pi \omega_0 n}{\omega_0}\right) + b_n \sin\left(\omega_0 n t + \frac{2\pi \omega_0 n}{\omega_0}\right) \right]$$

$$= a_0 + \sum_{n=1}^{k} \left[a_n \cos(\omega_0 n t + 2\pi n) + b_n \sin(\omega_0 n t + 2\pi n) \right]$$

$\because$ $\cos(2\pi n + \theta) = \cos\theta$.

also, $\sin(2\pi n + \theta) = \sin\theta$.

Then,

$$x(t) = a_0 + \sum_{n=1}^{k} (a_n \cos \omega_0 n t + b_n \sin \omega_0 n t).$$

$$x(t) = x(t) \quad \text{proved.}$$

Thus, we have, the infinite series of sine and cosine terms of frequencies $0, \omega_0, 2\omega_0, \ldots k\omega_0$ is known as trigonometric form of Fourier series.

i.e.,

$$x(t) = \sum_{n=0}^{\infty} a_n \cos \omega_0 n t + b_n \sin \omega_0 n t \quad \text{--- (1)}$$

where

~~$a_0, a_1, a_2, \ldots$~~

a_0,

a_n and b_n are called constant. The coefficient a_0 is called the DC component and $a_1 \cos \omega_0 t + b_1 \sin \omega_0 t$ is called first harmonic and so on.

Evaluation of Fourier coefficients of the Trigonometric Fourier Series

Calculation of a_0 To evaluate a_0, integrate $x(t)$ ~~over~~ on the both sides of $x(t)$ over one period t_0 to t_0+T.

$$\therefore \int_{t_0}^{t_0+T} x(t) = \int_{t_0}^{t_0+T} a_0 + \int_{t_0}^{t_0+T} \left[\sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]\right]$$

$$= a_0[t_0 + T - t_0] + \int_{t_0}^{t_0+T} \left[\sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right].$$

$$= a_0 T + \sum a_n \int_{t_0}^{t_0+T} \cos n\omega_0 t + b_n \int_{t_0}^{t_0+T} \sin n\omega_0 t$$

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 T + 0 + 0$$

$$\therefore a_n \int_{t_0}^{t_0+T} \cos n\omega_0 t = 0$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\cdot dt$$

∵ The net area of sinusoidal wave over complete period is 0.

Thus,

$$\boxed{a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\cdot dt}$$

Calculation of a_n

∴ we know that

$$x(t) = a_0 + \int_{t_0}^{t_0+T} \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

On multiplying on the both side of ~~above equation we get~~ the equation ① by $\cos m\omega_0 t$ we get:

$$\int_{t_0}^{t_0+T} x(t)\cdot \cos m\omega_0 t\,dt = a_0 \int_{t_0}^{t_0+T} \cos m\omega_0 t\,dt + \sum a_n \int_{t_0}^{t_0+T} \cos n\omega_0 t \cos m\omega_0 t + \int_{t_0}^{t_0+T} \sin n\omega_0 t \cdot \cos m\omega_0 t\,dt$$

The value of $\int_{t_0}^{t_0+T} \cos n\omega_0 t \cos m\omega_0 t \, dt = \begin{cases} 0 & ; \text{for } m \neq n \\ T/2 & \text{for } m = n \neq 0. \end{cases}$

$\int_{t_0}^{t_0+T} \sin n\omega_0 t \cdot \sin m\omega_0 t \, dt = \begin{cases} 0 & ; \text{for } m \neq n \\ T/2 & ; \text{for } m = n \neq 0. \end{cases}$

$\int_{t_0}^{t_0+T} \sin n\omega_0 t \cdot \cos m\omega_0 t \, dt = 0$ for all values of m and n.

substituting the value of above values in equation (1) we get

$$\int_{t_0}^{t_0+T} x(t) \cos m\omega_0 t \, dt = \underbrace{\int_{t_0}^{t_0+T} a_0 \cos m\omega_0 t \, dt}_{0} + \sum_{n=1}^{\infty} \int_{t_0}^{t_0+T} a_n \cos n\omega_0 t \cdot \cos m\omega_0 t + \underbrace{\int_{t_0}^{t_0+T} \sum_{n=1}^{\infty} \sin n\omega_0 t \cos m\omega_0 t}_{0}$$

$$= 0 + \sum_{n=1}^{\infty} \int_{t_0}^{t_0+T} (a_n \cos n\omega_0 t \cdot \cos m\omega_0 t) \, dt$$

$$\int_{t_0}^{t_0+T} x(t) \cdot \cos m\omega_0 t = \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} (\cos n\omega_0 t \cdot \cos m\omega_0 t) \, dt$$

If $m = n$

$$= a_m \cdot T/2$$

$$\therefore a_m = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cdot \cos m\omega_0 t$$

$$\therefore a_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos n\omega_0 t$$

Calculation of b_n

Multiply equation (I) by $\sin m\omega_0 t$ on the both sides we get,

$$\therefore \int_{t_0}^{t_0+T} x(t)\cdot \sin m\omega_0 t\, dt = \int_{t_0}^{t_0+T} a_0 \sin m\omega_0 t + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos n\omega_0 t \cdot \sin m\omega_0 t\, dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \sin n\omega_0 t \cdot \sin m\omega_0 t\, dt$$

On substituting the value of first and (II) second term equal to 0 because the area under these integrals is equal to 0.

$$\therefore \int_{t_0}^{t_0+T} x(t)\cdot \sin m\omega_0 t\, dt = 0 + 0 + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \sin n\omega_0 t \cdot \sin m\omega_0 t\, dt$$

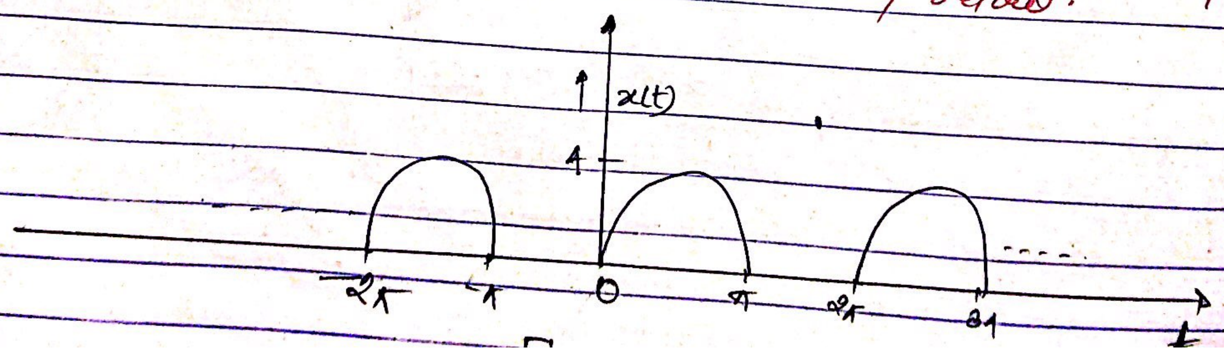
$$\int_{t_0}^{t_0+T} x(t)\cdot \sin m\omega_0 t\, dt = b_m T/2 \qquad [\text{at } m = n]$$

$$\therefore b_m = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cdot \sin n\omega_0 t\, dt \qquad [\because m = n]$$

Then,

$$\boxed{b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cdot \sin n\omega_0 t\, dt}$$

Q. Find the Fourier series expansions of the half wave rectified sine wave shown below.



② COSINE REPRESENTATION (ALTERNATE FORM OF THE TRIGONOMETRIC Representation)

The trigonometric Fourier series of x(t) contains sine and cosine terms of the same frequency.

i.e
$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$= a_0 + \sum_{n=1}^{\infty}\left[\sqrt{a_n^2+b_n^2}\left\{\left(\frac{a_n}{\sqrt{a_n^2+b_n^2}}\right)\cos n\omega_0 t + \left(\frac{b_n}{\sqrt{a_n^2+b_n^2}}\right)\sin n\omega_0 t\right\}\right]$$

Let $a_0 = A_0$

$A_n = \sqrt{a_n^2+b_n^2}$

$\cos\theta_n = \frac{a_n}{\sqrt{a_n^2+b_n^2}}$

$\sin\theta_n = \frac{-b_n}{\sqrt{a_n^2+b_n^2}}$

On substituting the value of these in above equation we get.

$$x(t) = A_n + \sum_{n=1}^{\infty}\left[A_n \cdot \cos\theta_n \cdot \cos n\omega_0 t + (-\sin\theta_n)\cdot \sin n\omega_0 t\right]$$

$$= A_n + \sum_{n=1}^{\infty}\left(A_n\left\{\cos\theta_n \cdot \cos n\omega_0 t - \sin\theta_n \sin n\omega_0 t\right\}\right)$$

$$= A_0 + \sum_{n=1}^{\infty} A_n\left\{\cos\theta_n t \cdot \cos n\omega_0 t - \sin\theta_n t \cdot \sin n\omega_0 t\right\}$$

$\because \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cdot \cos(n\omega_0 t + \omega_0 t)$$

So,
$$x(t) = A_0 + \sum_{n=1}^{\infty}\left[A_n \cos(n\omega_0 t + \theta_n)\right]$$
$[\because \omega_0 t = \theta_n]$

This form is also known as Harmonic form fourier series or polar form Fourier series.

The number A_n represents the amplitude coefficient or harmonic amplitudes or spectral amplitude.

θ_n represents the phase coefficients or phase angle or spectral phase of the fourier series.

Harm
Dispense

WAWE SYMMETRY

If the periodic signal x(t) has some type of symmetry, then some of the trigonometric Fourier series coefficients may be equal to 0 and the calculation of the coefficients becomes simple.
There are following four type of symmetry a function x(t) can have.

i) even symmetry
ii) Odd symmetry
iii) Half wave symmetry
iv) Quarter wave symmetry.

* If x(t) has even symmetry (also called mirror symmetry) then $b_n = 0$, and only a_0 and a_n is to be evaluated.

* If x(t) has odd symmetry (also called rotation symmetry), then $a_0 = 0$ and $a_n = 0$. Only b_n is evaluated.

* If x(t) has half wave symmetry, then $a_0 = 0$ and only odd harmonics ~~symmetry~~ exist.

* If x(t) has quarter wave symmetry, then $a_0 = 0$, and either only a_n or b_n exists only for odd value of n.

As we know that,
The even signal is given by
$$x(t) = x(-t)$$
and odd signal is given by
$$x(t) = -x(-t)$$
As we know that any function x(t) can be expressed as sum of even part and odd part of the signal.
$$x(t) = x_e(t) + x_o(t)$$
The even part is given by
$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$
and odd part is given by
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Here, we choose the interval of integration from $-T/2$ to $T/2$ instead of form t_0 to t_0+T.

Remember,

i) Odd function × Odd function = even function.

ii) even function × even function = even function

iii) Odd function × even function = Odd function.

* For even function $x_e(t)$

$$x_e(t) = \int_{-T/2}^{T/2} x_e(t)\cdot dt = 2\int_{0}^{T/2} x_e(t)\cdot dt$$

For, odd function $x_o(t)$.

$$\int_{-T/2}^{T/2} x_o(t)\cdot dt = 0.$$

Based on these relation,

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \cos n\omega_0 t\, dt \quad \text{--- (I)}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \sin n\omega_0 t\, dt \quad \text{--- (II)}$$

On putting the value of $x(t) = x_e(t) + x_o(t)$ in above eqn we get,

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} [x_e(t) + x_o(t)]\, \cos n\omega_0 t\, dt$$

$$\therefore a_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} x_e(t)\cdot \cos n\omega_0 t\, dt + \int_{-T/2}^{T/2} x_o(t)\cdot \cos n\omega_0 t\, dt\right]$$

$$b_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} x_e(t)\cdot \sin n\omega_0 t\, dt + \int_{-T/2}^{T/2} x_o(t)\cdot \sin n\omega_0 t\, dt\right]$$

EVEN or MIRROR SYMMETRY.

even or mirror symmetry: A function x(t) is said to have even or mirror symmetry if

$$x(t) = x(-t) \quad -(A)$$

If a constant is added, then even nature of the function still persists.

$$\therefore \quad a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \cos n\omega_0 t\, dt$$

as $x(t) = x_e(t)$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x_e(t)\cdot \cos n\omega_0 t\, dt$$

$$a_n = \frac{4}{T}\int_{0}^{T/2} x(t)\cdot \cos n\omega_0 t\, dt$$

as $x(t) = x_e(t) + x_o(t) = x_e(t)$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x_e(t)\, dt = \frac{2}{T}\int_{0}^{T/2} x_e(t)\cdot dt$$

and

$$b_n = \frac{4}{T}\int_{0}^{T/2} x_e(t) \sin n\omega_0 t\, dt = 0$$

because $x_e(t) \sin n\omega_0 t\, dt$ is an odd function and the area under a complete cycle is 0.

ODD or ROTATION SYMMETRY

* If a constant is added to an odd function, then the odd nature of the function is removed.

EVEN or MIRROR SYMMETRY.

even or mirror symmetry: A function $x(t)$ is said to have even or mirror symmetry if

$$x(t) = x(-t) \quad \text{--- (A)}$$

If a constant is added, then even nature of the function still persists.

$\therefore$ $$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t).\cos n\omega_0 t\, dt$$

as $x(t) = x_e(t)$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x_e(t).\cos n\omega_0 t\, dt$$

$$a_n = \frac{4}{T}\int_{0}^{T/2} x(t).\cos n\omega_0 t\, dt$$

as $x(t) = x_e(t) + x_o(t) = x_e(t)$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t).dt$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x_e(t).dt = \frac{2}{T}\int_{0}^{T/2} x_e(t).dt$$

and

$$b_n = \frac{4}{T}\int_{0}^{T/2} x_e(t) \sin n\omega_0 t\, dt = 0$$

because $x_e(t) \sin n\omega_0 t\, dt$ is an odd function, and the area under a complete cycle is 0.

ODD or ROTATION SYMMETRY

* If a constant is added, to an odd function, then the odd nature of the function is removed.

So,

$$a_0 = 0$$

$$a_n = 0$$

also,

$$b_n = \frac{4}{T}\left[\int_0^{T/2} x(t)\sin n\omega_0 t\, dt\right]$$

HALF WAVE SYMMETRY

Half wave symmetry: A periodic signal $x(t)$ which satisfy the following condition

$$x(t) = -x\left(t \pm T/2\right)$$

is said to have half wave symmetry.

This function is neither purely odd nor purely even. For such functions, $a_0 = 0$.

The Fourier series expansion of such type of periodic signal contains odd harmonics only i.e $\omega_0, 3\omega_0, 5\omega_0$ ---- etc.

As we know that,

$x(t)$ has only odd harmonics terms,

then for n = even

$$a_n = b_n = 0$$

when n = odd,

Then

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cdot \cos n\omega_0 t\, dt$$

$$b_n = 4/T\int_0^{T/2} x(t)\cdot \sin n\omega_0 t\, dt$$

The figure of half wave symmetry is shown below

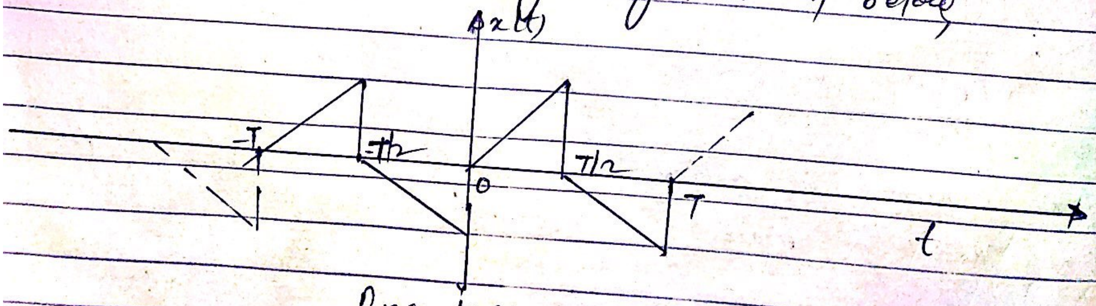


fig: Half wave symmetry

QUARTER WAWE SYMMETRY

Quarter wave symmetry:- A function $x(t)$ is said to have quarter wave symmetry if

$$x(t) = x(-t)$$

or

$$x(t) = -x(-t)$$

and also,

$$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

For a function $x(t)$ which has either even symmetry or odd symmetry along with half wave symmetry is said to quarter wave symmetry.

The below figure shows the example of figure of quarter wave functions.

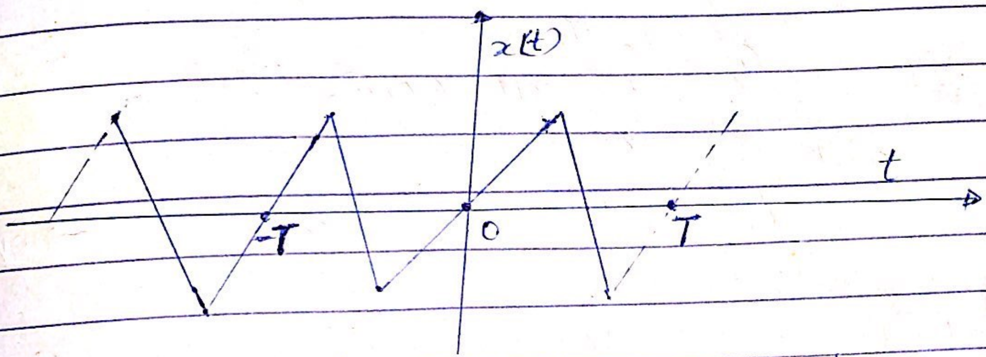


Fig: Quarter wave symmetry

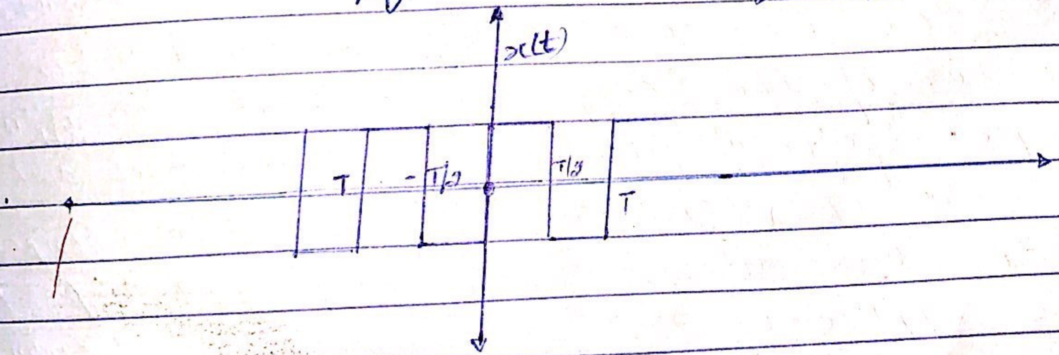
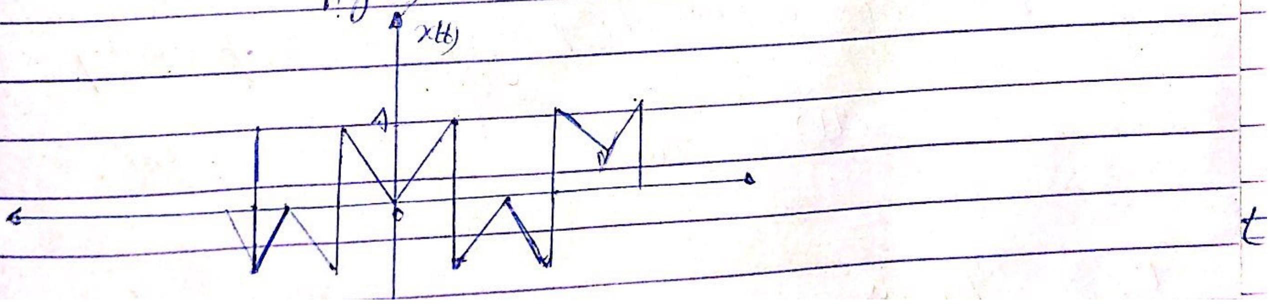


Fig (b)



The following two cases are discussed below for the waveform having quarter wave symmetry.

Case-I

$$x(t) = -x(-t) \quad \text{and}$$
$$x(t) = -x(t \pm T/2)$$

Then the coefficient of fourier series are

$$a_0 = 0$$
$$a_n = 0$$
$$b_n = \frac{8}{T}\int_0^{T/4} x(t)\cdot \sin n\omega_0 t \, dt \quad [\text{when } n = \text{odd}]$$

Case-II

$$x(t) = x(-t)$$
$$\text{and } x(t) = -x(t + T/2)$$

Then, the coefficient of Fourier series are,

$$a_0 = 0$$
$$a_n = \frac{8}{T}\int_0^{T/2} x(t)\cdot \cos n\omega_0 t \, dt$$

and

$$b_n = 0 \quad [\text{when } n = \text{even}]$$

EXPONENTIAL FOURIER SERIES

It is most widely used form of fourier series. In this, the function $x(t)$ is expressed as a weighted sum of the complex exponential form function. The complex exponential form is more general and usually more convenient and more compact. So, it is used almost exclusively, and it finds extensive application in communication theory.

∴ The cosine form of trigonometric fourier series

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n) \quad \text{—— (1)}$$

Using Euler's identity,

$$\cos(n\omega_0 t + \phi_n) = \frac{e^{j(n\omega_0 t + \phi_n)} + e^{-j(n\omega_0 t + \phi_n)}}{2}$$

On substituting the values of $\cos(n\omega_0 t + \theta_n)$ in equation (1) we get.

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \left(\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}}{2} \right)$$

$$= A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j(n\omega_0 t + \theta_n)} + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{-j(n\omega_0 t + \theta_n)}$$

$$= A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j(n\omega_0 t + \theta_n)} + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{-j(n\omega_0 t + \theta_n)}$$

$$= A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j\theta_n} e^{jn\omega_0 t} + \sum \frac{A_n}{2} \cdot e^{-jn\omega_0 t} \cdot e^{-j\theta_n}$$

$$= A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{j\theta_n} \right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{-j\theta_n} \right) e^{-jn\omega_0 t}$$

Let $n = -k$, put this in the second summation we get

$$x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j\theta_n} e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty} \ldots$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{j\theta_n} \right) \cdot e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty} \frac{A_k}{2} e^{j(\theta_k)} \cdot e^{-jn\omega_0 t}$$

$$A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} \cdot e^{j\theta_n} \cdot e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty} \frac{A_k}{2} e^{(j\theta_k)} \cdot e^{jk\omega_0 t}$$

$$= A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} \cdot e^{j\theta_n} \right) e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty} \left(\frac{A_k}{2} e^{(j\theta_k)} \right) \cdot e^{jk\omega_0 t} \quad \text{— (11)}$$

Now, Comparing the above two equations we get

$A_n = A_k$; $(-\theta_n) = \theta_k$ $\quad n>0, \; k<0.$

Let define.

$$C_0 = A_0 ; \quad C_n = \frac{A_n}{2} e^{j\theta_n}, \quad n>0$$

put the values in equation (2) we get,

$$\therefore \quad x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j\theta_n} \cdot e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty} \frac{A_k}{2} e^{(j\theta_k)} \cdot e^{jk\omega_0 t}$$

$$= C_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{j\theta_n}\right) e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty}\left(\frac{A_n}{2} e^{j\theta_n}\right) e^{jn\omega_0 t}$$

put $C_n = \frac{A_n}{2} e^{j\theta_n}$

so,

$$x(t) = \sum_{n=-\infty}^{n=+\infty} C_n \cdot e^{jn\omega_0 t}$$

Thus,

$$\boxed{x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}}$$

$\therefore t = 0$

$\therefore A_0 = C_0 = 0.$

This is the exponential form of Fourier series. This is also known as synthesis equation.

Determination of the Coefficient of exponential Fourier series.

$$\therefore \quad x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t} \quad ; \; \omega_0 = \frac{2\pi}{T}$$

Now, on multiplying by $e^{-jk\omega_0 t}$ and integrating over one period we get:-

$$\int_{t_0}^{t_0+T} x(t) \cdot e^{-jk\omega_0 t} \, dt = \int_{t_0}^{t_0+T} \sum_{-\infty}^{\infty} \left(C_n e^{jn\omega_0 t}\right) \cdot e^{-jk\omega_0 t} \, dt$$

$$= \sum_{n=-\infty}^{\infty} C_n \int_{t_0}^{t_0+T} e^{jn\omega_0 t} \cdot e^{-jk\omega_0 t} \, dt$$

Here,

$$= \sum_{n=-\infty}^{\infty} C_n \int_{t_0}^{t_0+T} e^{(n-k)j\omega_0 t} \, dt =$$

$\therefore$ we know that,

$$\int_{t_0}^{t_0+T} \left(e^{jn\omega_0 t} \cdot e^{-jk\omega_0 t}\right) dt = \begin{cases} 0 ; & n \neq k \\ T ; & n = k \end{cases}$$

At; $n=k$ then

$$\int_{t_0}^{t_0+T} x(t) \, e^{-jk\omega_0 t} \, dt = T C_k .$$

$$\therefore \quad C_k = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) \cdot e^{-jk\omega_0 t} \, dt$$

or

fourier coefficient $\rightarrow$

$$C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) \cdot e^{-jn\omega_0 t} \, dt$$

The above equation is also called / known as analysis equation.

and, At $n=0$,

$$C_0 = A_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) \cdot dt$$

* here, it represents a complex spectrum, it has both magnitude and phase spectrum.

1. The magnitude line spectrum is always an even function of
2. The phase line spectrum is always an odd function of n.